C.3

微積分(II) 期中考試 (2002/4/11)

Problem 1. (10pts)
Find the sum of the convergent series $\sum_{n=1}^{\infty} \arctan \frac{1}{n^2+n+1}$.

Problem 2. (50pts) Determine whether the series is convergent or divergent.

(a) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3+1}}$.

(b) $\sum_{n=3}^{\infty} \frac{1}{(\ln n)^{\ln \ln n}}$.

(c) $\sum_{n=1}^{\infty} (-1)^{n+1} \operatorname{arccot} n$.

(d) $\sum_{n=1}^{\infty} (-1)^{n+1} n \arctan \frac{1}{n^2}$.

(e) $\sum_{n=1}^{\infty} (-1)^n (n \sin \frac{1}{n} - 1)$.

Problem 3. (10pts) Show that $\ln(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \cdots$, for $|x| < 1$.

Problem 4. (10pts) Find the power series expansion of $\ln(x^2+x+1)$, and determine the radius of convergence of the series.

Problem 8. (20pts)
Define the function g as follows: $g(x) = \exp(-1/x^2)$ if $x \neq 0$, $g(0) = 0$. Show that $g^{[n]}(0)$ exists and equals 0 for every integer n. Prove that the Maclaurin series for g is not equal to g in any neighborhood of 0.

Good Luck !!